

# The RN/CFT Correspondence Revisited

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## Abstract

We reconsidered the quantum gravity description of the near horizon extremal Reissner-Nordström black hole in the viewpoint of the  $\text{AdS}_2/\text{CFT}_1$  correspondence. We found that, for pure electric case, the right moving central charge of dual 1D CFT is  $6Q^2$  which is different from the previous result  $6Q^3$  of left moving sector obtained by warped  $\text{AdS}_3/\text{CFT}_2$  description. We discussed the discrepancy in these two approaches and examined novel properties of our result.

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## I. INTRODUCTION

Searching for the full quantum theory of gravity is still a tough problem. Although the holographic principle, in particular the explicit realization for AdS/CFT correspondence, open a new visual angle to solve this problem [1, 2, 3, 4, 5], there are so far only few well-understood examples such as the  $\text{AdS}_5/\text{CFT}_4$  [3] and  $\text{AdS}_3/\text{CFT}_2$  [6] dualities. Recently, an interesting progress has been made along this direction for extremal Kerr black holes—the Kerr/CFT correspondence [7], in which the central charge of the dual chiral CFT (left movers) can be identified by studying the asymptotic symmetry of the near horizon extremal Kerr black hole (NHEK) geometry. The NHEK geometry contains an  $\text{AdS}_2$  factor with an  $S^1$  bundle assembled as a warped  $\text{AdS}_3$  geometry which involves  $SL(2, \mathbb{R})_R \times U(1)_L$  symmetry. This consequence indicates that a new duality, warped  $\text{AdS}_3/\text{CFT}_2$  correspondence, may exist. Soon after, the Kerr/CFT correspondence has been generalized into many other spacetimes which contain a warped  $\text{AdS}_3$  structure [8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18,

19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32]. However, the initial work [7] did not address more information about the dual CFT in addition to the central charge. Therefore, new schemes should be developed to understand the properties of the CFT and dynamics behind this duality. A promising suggestion is to construct the stress tensor of the dual CFT (2D or 1D) and study the corresponding properties from the conserved charges [33, 34, 35].

Besides, it is also interesting to ask whether we can find the CFT description for extremal non-rotating black holes and for near extremal generalization. Indeed, the CFT dual of extremal Reissner-Nordström (RN) black hole has been studied by using the analogous treatment in Kerr/CFT correspondence [36, 37]. In that approach, the off-diagonal part of the metric, the  $U(1)$  bundle of warped  $\text{AdS}_3$ , was recovered from the gauge field potential by uplifting the RN black hole into 5D gravity. Then the left-moving central charge,  $c_L = 6Q^3$ , and temperature,  $T_L = 1/2\pi Q$ , were computed [36, 37] which reproduce the black hole entropy by the Cardy formula. However, these results have unlike properties comparing with Kerr black hole case. For example, the CFT temperature is dependent on the black hole parameter unlike the constant temperature  $T_L = 1/2\pi$  for Kerr black hole. Moreover, the central charge (left movers) in the Kerr/CFT correspondence  $c_L = 12J = 6l^2/G_4$  is related to the angular momentum,  $J$ , or equivalently to the square of the warped  $\text{AdS}_3$  radius,  $l$ , of the NHEK geometry [7]. While in the near horizon (near) extremal RN black hole geometry, the radius of  $\text{AdS}_2$  factor is given by the charge parameter  $q$ . Thus a naive but reasonable expectation is that the central charge corresponding to the RN black hole is proportional to  $q^2$  and the level-central charge relation  $h_L = c_L/24$  for  $\text{AdS}_3$  could hold, then the CFT temperature is still a constant  $T_L = 1/2\pi$ . In this paper, we check such conjecture by considering the 2D effective action of 4D extremal RN black hole via a dimensional reduction. Analogous to the recent work by Castro and Larsen [35] on the near extremal Kerr black hole, we used the boundary counterterm method [38, 39] in the framework of the  $\text{AdS}_2/\text{CFT}_1$  correspondence to derive the “right-moving” central charge. Our result shows that the central charge of 1D CFT dual to the extremal RN black hole is  $c_R = 6q^2/G_4 = 6Q^2$ . Since there is no gravitational anomaly, it is reasonable to expect  $c_L = c_R$ , then using the Cardy formula, the entropy of extremal RN black hole is reproduced, which confirms the conjecture. Note that, if we identify  $c_L$  with  $c_R$ , then our result indicates a discrepancy compared with those obtained in [36, 37] from the warped  $\text{AdS}_3$  descriptions. Actually, there is a scaling ambiguity in both  $\text{AdS}_2$  and warped  $\text{AdS}_3$  descriptions to determine the

central charge dual to RN black hole. We will discuss the details on issue in the section of Conclusion and Discussion.

This paper is organized as follows. In section II, we briefly review the near horizon near extremal RN black hole, and then in section III, we study the stress tensor and current of the dual 1D CFT by using the boundary counterterm method. In section IV, by imposing the appropriate asymptotic boundary conditions and combining of the diffeomorphism transformation together with the gauge transformation, we obtain the central charge of the dual CFT and calculate the conserved charges. In section V, we give another derivation to support our result. Finally, we come to the conclusion and discussion. Moreover, the derivation of the central charge for dyonic RN black holes is briefly summarized in the appendix A.

## II. REVIEW OF THE NEAR EXTREMAL RN BLACK HOLE

The action of the 4D Einstein-Maxwell theory

$$I = \frac{1}{16\pi G_4} \int d^4x \sqrt{-g_4} (R_4 - F^2), \quad (1)$$

admits uniquely spherically symmetric electro-vacuum solution—the Reissner-Nordström (RN) black hole. The explicit expressions for the electric charged RN black hole are

$$\begin{aligned} ds^2 &= - \left( 1 - \frac{2m}{r} + \frac{q^2}{r^2} \right) dt^2 + \frac{dr^2}{1 - \frac{2m}{r} + \frac{q^2}{r^2}} + r^2 d\Omega_2^2, \\ A &= \frac{q}{r} dt, \quad F = \frac{q}{r^2} dt \wedge dr, \end{aligned} \quad (2)$$

in which two parameters  $m, q$  representing the mass and electric charge.<sup>1</sup> The general RN black hole has two horizons and the corresponding outer and inner horizon radii are

$$r_{\pm} = m \pm \sqrt{m^2 - q^2}. \quad (3)$$

The region enclosed by outer horizon behaves like a thermodynamical system with Hawking temperature and Bekenstein-Hawking entropy

$$T_H = \frac{\kappa}{2\pi} = \frac{r_+ - r_-}{4\pi r_+^2}, \quad S_{BH} = \frac{A_+}{4G_4} = \frac{\pi r_+^2}{G_4}, \quad (4)$$

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<sup>1</sup> The parameters  $m$  and  $q$  have dimension of length and the ADM mass and the electric charge (dimensionless) are given by  $M = m/G_4, Q = q/\sqrt{G_4}$ .

where  $\kappa$  and  $A_+$  are the surface gravity and area of the outer horizon, respectively.

The near horizon geometry of extremal RN black holes, i.e.  $m = q$  (assuming  $q > 0$ ), includes a  $AdS_2$  structure revealing existence of a CFT description. Technically, one can obtain the near horizon geometry of near extremal RN black hole by taking the following limits of  $\varepsilon \rightarrow 0$

$$r \rightarrow q + \varepsilon\rho, \quad t \rightarrow \frac{\tau}{\varepsilon}, \quad m = q + \frac{\varepsilon^2 b^2}{2q}, \quad (5)$$

and the near horizon solution is [40, 41, 42]

$$\begin{aligned} ds^2 &= -\frac{\rho^2 - b^2}{q^2} d\tau^2 + \frac{q^2}{\rho^2 - b^2} d\rho^2 + q^2 d\Omega_2^2, \\ A &= -\frac{\rho}{q} d\tau, \quad F = \frac{1}{q} d\tau \wedge d\rho. \end{aligned} \quad (6)$$

Here the radius of  $AdS_2$  is given by the charge parameter  $q$  and the combination  $\varepsilon b$  labels the derivation from the extremality. Thus, the black hole entropy can be expanded as

$$S_{BH} = \frac{\pi}{G_4} (q^2 + 2q\varepsilon b + \mathcal{O}(\varepsilon^2 b^2)). \quad (7)$$

For the extremal case, i.e.  $b = 0$ , the black hole entropy,  $S_{BH} = \pi q^2/G_4$ , is expected to match the entropy of the dual CFT calculated from the Cardy formula

$$S_{CFT} = 2\pi \sqrt{\frac{c_L h_L}{6}}. \quad (8)$$

The central charge and temperature, in the extremal limit, have been calculated in the warped  $AdS_3/CFT_2$  picture [36, 37]

$$c_L = 6Q^3, \quad T_L = \frac{1}{2\pi Q} \quad \Rightarrow \quad h_L = \frac{\pi^2 T^2 c}{6} = \frac{Q}{4}. \quad (9)$$

However, the relations  $c_L \propto Q^3$  and  $T_L \propto 1/Q$  seem very unnatural. Inspired from the Kerr/CFT results [35], one may expect that the central charge is proportional to the square of  $AdS_2$  radius is a general picture, which implies  $c_L \propto q^2$  for RN black hole. Moreover, if we further assume the relations  $h_L = c_L/24$  is still hold (like the 2D CFT case), then the matching of black hole and CFT entropies gives

$$c_L = \frac{6q^2}{G_4} = 6Q^2, \quad h_L = \frac{q^2}{4G_4} = \frac{1}{4}Q^2, \quad T_L = \frac{1}{2\pi}. \quad (10)$$

These expected results involve physically promising properties. In the following sections, we will explicitly calculate the right-moving central charge to verify our speculation eq.(10) based on the expectation  $c_L = c_R$  (since there is no gravitational anomaly).

### III. THE 2D EFFECTIVE THEORY

We study the CFT dual description for the RN black hole by considering a 2D effective action dimensionally reduced from the 4D Einstein-Maxwell action (1). By assuming proper ansatz for metric and gauge potential

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu + q^2 e^{-2\psi} d\Omega_2^2, \quad A = A_\mu dx^\mu, \quad (11)$$

we can straightforwardly compute the following geometric quantities

$$R_4 = R_2 + \frac{2}{q^2} e^{2\psi} - 2e^{2\psi} \nabla^2 e^{-2\psi} + 2e^{2\psi} \nabla_\mu e^{-\psi} \nabla^\mu e^{-\psi}, \quad (12)$$

and

$$\sqrt{-g_4} = q^2 e^{-2\psi} \sin \theta \sqrt{-g_2}. \quad (13)$$

Therefore, the 2D effective action is

$$I = \frac{q^2}{4G_4} \int d^2x \sqrt{-g_2} \left( e^{-2\psi} R_2 + \frac{2}{q^2} + 2\nabla_\mu e^{-\psi} \nabla^\mu e^{-\psi} - e^{-2\psi} F^2 \right), \quad (14)$$

where  $R_2$  is the 2D Ricci scalar associated with  $g_{\mu\nu}$ ,  $\psi$  is the dilaton field and  $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$  is the  $U(1)$  gauge field strength. The corresponding equations of motion for constant  $\psi$ , which is a consistent truncation, are (note that  $R_{\mu\nu} = \frac{1}{2} R g_{\mu\nu}$  for 2D geometry)

$$\begin{aligned} R_2 - F^2 &= 0, \\ \nabla_\mu F^{\mu\nu} &= 0, \\ \frac{1}{2} g_{\mu\nu} \left( \frac{1}{q^2} e^{2\psi} - \frac{1}{2} F^2 \right) + F_{\mu\alpha} F_\nu{}^\alpha &= 0. \end{aligned} \quad (15)$$

From the above equations we get

$$R_2 = F^2 = -\frac{2}{q^2} e^{2\psi}. \quad (16)$$

Therefore the radius of local  $\text{AdS}_2$  solution, with constant  $\psi$ , is

$$\ell_{\text{AdS}} = q e^{-\psi}. \quad (17)$$

For the near horizon (near) extremal RN black hole,  $e^{-\psi} = 1$ . By a suitable choice of gauges, we assume the 2D general solution also takes the form [35]

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = e^{-2\psi} dr^2 + \gamma_{tt}(t, r) dt^2, \quad A = A_t(t, r) dt, \quad (18)$$

then the equations of motion reduce to

$$\gamma_{tt} = -q^2(\partial_r A_t)^2, \quad 2\gamma_{tt}\partial^2\gamma_{tt} - (\partial_r\gamma_{tt})^2 - 4q^{-2}\gamma_{tt}^2 = 0. \quad (19)$$

The general solution is characterized by a free time-dependent function

$$\gamma_{tt} = -\frac{q^2}{16} [e^{r/q} - f(t)e^{-r/q}]^2 \sim -\frac{q^2}{16} e^{2r/q}, \quad (20)$$

$$A_t = -\frac{q}{4} e^{r/q} \left[ 1 - \sqrt{f(t)} e^{-r/q} \right]^2 \sim -\frac{q}{4} e^{r/q}. \quad (21)$$

The special solution  $f = 0$  corresponds to extremal case, and for the near extremal case  $f = 4b^2/q^4$ . Moreover, the extrinsic curvature at boundary, i.e.  $r \rightarrow \infty$ , is

$$K = \lim_{r \rightarrow \infty} \frac{1}{2} \gamma^{tt} n^\mu \partial_\mu \gamma_{tt} = \frac{1}{q} e^\psi, \quad (22)$$

where  $n^r = \sqrt{g^{rr}} = e^\psi$ .

### A. Boundary counterterms

In order to obtain the renormalized finite boundary stress tensor of the dual 1D CFT, an effective way is to add suitable boundary counterterms, for gravity and matter fields, into the action [38, 39]. In the asymptotic  $\text{AdS}_2$  case considered here, the geometric counterterm is just the Gibbons-Hawking term and the rest boundary counterterm comes from the contribution of gauge field, i.e.

$$I_{\text{bndy}} = I_{\text{GH}} + I_{\text{counter}}, \quad (23)$$

where

$$I_{\text{GH}} = \frac{q^2}{2G_4} \int dt \sqrt{-\gamma} e^{-2\psi} K, \quad (24)$$

$$I_{\text{counter}} = \frac{q^2}{2G_4} \int dt \sqrt{-\gamma} (\alpha e^{-\psi} + \beta e^{-\psi} A_a A^a), \quad (25)$$

and  $\alpha, \beta$  are constants to be determined. The variation of full action takes the form

$$\delta I = (\text{bulk terms}) + \int dt \sqrt{-\gamma} (\pi_{ab} \delta \gamma^{ab} + \pi_\psi \delta \psi + \pi^a \delta A_a), \quad (26)$$

where

$$\begin{aligned} \pi_{tt} &= -\frac{q^2}{4G_4} (\alpha e^{-\psi} \gamma_{tt} + \beta e^{-\psi} A_a A^a \gamma_{tt} - 2\beta e^{-\psi} A_t A_t) \sim -\frac{q^2}{4G_4} e^{-\psi} \gamma_{tt} (\alpha + \beta), \\ \pi_\psi &= -\frac{q^2}{2G_4} (2e^{-2\psi} K + \alpha e^{-\psi} + \beta e^{-\psi} A_a A^a) \sim -\frac{q^2}{2G_4} e^{-\psi} \left( \frac{2}{q} + \alpha - \beta \right), \\ \pi^t &= \frac{q^2}{4G_4} (-4e^{-2\psi} n_\mu F^{\mu t} + 4\beta e^{-\psi} A^t) \sim \frac{q^2}{4G_4} e^{-\psi} \gamma^{tt} e^{r/q} (1 - \beta q). \end{aligned} \quad (27)$$

A well-defined variational principle requires all the coefficients  $\pi_{ab}$ ,  $\pi_\psi$  and  $\pi^a$  be finite. Similar to the Kerr black hole case considered in [35], the values of  $\alpha, \beta$  can be determined by ensuring vanishing leading terms of these three boundary momenta, and we obtain

$$\alpha = -\frac{1}{q}, \quad \beta = \frac{1}{q}. \quad (28)$$

## B. Boundary currents

The boundary currents are defined by

$$\begin{aligned} T_{ab} &= -\frac{2}{\sqrt{-\gamma}} \frac{\delta I}{\delta \gamma^{ab}} = -2\pi_{ab}, \\ J^a &= -\frac{1}{\sqrt{-\gamma}} \frac{\delta I}{\delta A_a} = -\pi^a, \end{aligned} \quad (29)$$

and their corresponding components are

$$\begin{aligned} T_{tt} &= -\frac{q}{2G_4} e^{-\psi} (\gamma_{tt} + A_t A_t) = \frac{q^3}{8G_4} e^{-\psi} e^{r/q} \sqrt{f} \left(1 - \sqrt{f} e^{-r/q}\right)^2, \\ J_t &= -\frac{q}{G_4} e^{-\psi} (A_t - q e^{-\psi} n^\mu F_{\mu t}) = -\frac{q^2}{2G_4} e^{-\psi} \sqrt{f} \left(1 - \sqrt{f} e^{-r/q}\right). \end{aligned} \quad (30)$$

The exact solution corresponding to near horizon of near extremal RN black hole is

$$\begin{aligned} \gamma_{tt} &= -\frac{q^2}{16} \left( e^{r/q} - \frac{4b^2}{q^4} e^{-r/q} \right)^2, \\ A_t &= -\frac{q}{4} e^{r/q} \left( 1 - \frac{2b}{q^2} e^{-r/q} \right)^2, \end{aligned} \quad (31)$$

and consequently the associated currents are

$$\begin{aligned} T_{tt} &= \frac{qb}{4G_4} e^{-\psi} \left( e^{r/q} - \frac{4b}{q^2} + \frac{4b^2}{q^4} e^{-r/q} \right), \\ J_t &= -\frac{q^2}{G_4} e^{-\psi} \left( \frac{b}{q^2} - \frac{2b^2}{q^4} e^{-r/q} \right). \end{aligned} \quad (32)$$

Like the near extremal Kerr black hole case, for  $r \gg q$ , the  $T_{tt}$  is still divergent.

## IV. ASYMPTOTIC SYMMETRIES AND CONSERVED CHARGES

As has been point out in [35, 43], for a chosen asymptotic symmetry of spacetime, the gauge fields need not only be diffeomorphism invariant but also be gauge invariant. We can



see that the combination of these two transformations indeed give the central charge we expected. The asymptotic boundary conditions are

$$\delta_\epsilon g_{rr} = 0, \quad \delta_\epsilon g_{tr} = 0, \quad \delta_\epsilon g_{tt} = 0 \cdot e^{2r/q} + \dots, \quad (33)$$

which lead to the allowed transformations

$$\epsilon^r = -q \partial_t \xi(t), \quad \epsilon^t = \xi(t) + 8e^{-2\psi} (e^{2r/q} - f(t))^{-1} \partial_t^2 \xi(t). \quad (34)$$

The boundary metric is transformed as

$$\delta_\epsilon \gamma_{tt} = q^2 (1 - f e^{-2r/q}) \left[ \frac{1}{4} f \partial_t \xi + \frac{1}{8} \xi \partial_t f - e^{-2\psi} \partial_t^3 \xi \right]. \quad (35)$$

However, the result

$$\delta_\epsilon A_r = 4 \partial_t^2 \xi e^{-2\psi} e^{-r/q} \left( 1 + \sqrt{f} e^{-r/q} \right)^{-2}, \quad (36)$$

violates the gauge condition  $A_r = 0$ . So we need a compensated gauge transformation

$$\Lambda = 4q e^{-2\psi} e^{-r/q} \left( 1 + \sqrt{f} e^{-r/q} \right)^{-1} \partial_t^2 \xi, \quad (37)$$

such that

$$\delta_{\epsilon+\Lambda} A_r = \delta_\epsilon A_r + \partial_r \Lambda = 0. \quad (38)$$

Thus variation of the time component of gauge field is

$$\begin{aligned} \delta_{\epsilon+\Lambda} A_t &= \frac{q}{2} (1 - \sqrt{f} e^{-r/q}) \partial_t (\sqrt{f} \xi) + 2q e^{-2\psi} e^{-r/q} \partial_t^3 \xi \\ &= \frac{q}{2} \partial_t (\sqrt{f} \xi) + \frac{q}{2} e^{-r/q} \left( -f \partial_t \xi - \frac{1}{2} \xi \partial_t f + 4e^{-2\psi} \partial_t^3 \xi \right), \end{aligned} \quad (39)$$

and variation of the stress tensor is

$$\begin{aligned} \delta_{\epsilon+\Lambda} T_{tt} &= -\frac{q}{2G_4} e^{-\psi} (\delta_\epsilon \gamma_{tt} + 2A_t \delta_{\epsilon+\Lambda} A_t) \\ &= 2T_{tt} \partial_t \xi + \xi \partial_t T_{tt} \\ &\quad + \frac{q^3}{G_4} \left[ e^{-3\psi} \partial_t^3 \xi - \frac{1}{8} e^{-\psi} \sqrt{f} e^{r/q} \left( 1 + \sqrt{f} e^{-r/q} \right) \partial_t \xi \right] (1 - \sqrt{f} e^{-r/q}). \end{aligned} \quad (40)$$

The right-moving central charge can be read out from the following relation [44]

$$\delta_{\epsilon+\Lambda} T_{tt} = 2T_{tt} \partial_t \xi + \xi \partial_t T_{tt} - \frac{c}{12} L \partial_t^3 \xi, \quad (41)$$

here the normalization factor  $L$  of dimension of length is needed to ensure the central charge to be dimensionless. It turns out that a physically suitable value of the normalization is

proportional to the AdS radius  $L$  by a factor  $-2$ , namely  $L = -2\ell_{AdS} = -2qe^{-\psi}$ , due to the same factor appearing in the cosmological constant term in the effective action (14).<sup>2</sup> Actually, in the next section, we will see that this factor  $-2$  endorses the ground value for the level. Finally we can easily read out the right moving central charge for the extremal RN black hole ( $f = 0, \psi = 0$ )

$$c_R = \frac{3L^2}{2G_4} = \frac{6q^2}{G_4} = 6Q^2. \quad (42)$$

Since there is no gravitational anomaly, similar to the near extremal Kerr black hole case [35], it is reasonable to expect that  $c_L = c_R$ , This is just we have expected in eq.(10).

For the near horizon extremal RN black hole, the Noether charge generated by the gauge transformation is

$$\delta_\Lambda I = - \int dt \sqrt{-\gamma} \mathcal{J}^a \partial_a \Lambda, \quad (43)$$

therefore

$$\mathcal{J}^t = - \frac{\delta I_{\text{counter}}}{\sqrt{-\gamma} \delta A_t} = - \frac{q}{G_4} e^{-\psi} A^t, \quad (44)$$

and the associated charge is

$$Q_\Lambda = \sqrt{-\gamma} \mathcal{J}^t = \frac{qe^{-\psi}}{G_4} \frac{1 - \sqrt{f}e^{-r/q}}{1 + \sqrt{f}e^{-r/q}} \approx \frac{q}{G_4} e^{-\psi}, \quad (45)$$

which identical to the ADM energy for the extremal RN black hole.

The level  $k$  is defined as

$$\delta_\Lambda \mathcal{J}_t = \frac{k}{2} L \partial_t \Lambda, \quad (46)$$

and

$$\delta_\Lambda \mathcal{J}_t = - \frac{1}{2G_4} (2qe^{-\psi}) \partial_t \Lambda, \quad (47)$$

therefore

$$k = \frac{1}{G_4}. \quad (48)$$

Note that the presence of the gravitational constant  $G_4$  is due the the overall factor  $1/16\pi G_4$  in the matter part of the action (1). In the usual convention, this factor is absorbed in the Maxwell field strength and then the value is  $k = 1$ . Moreover, the normalization factor  $L$  in the definition of level ensures the ground value.

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<sup>2</sup> The effective AdS radius indeed is exactly the notion  $L$  defined in the action (50).

## V. ANOTHER DERIVATION

In this section we will check the validity of the central charge eq.(42) by comparing with the results in [43, 44]. Recall that our effective action (14) is consistently truncated, for  $\psi = 0$ , to

$$I = \frac{1}{4G_4} \int d^2x \sqrt{-g_2} \left[ q^2 \left( R_2 + \frac{2}{q^2} \right) - q^2 F^2 \right], \quad (49)$$

which is equivalent to the action considered in [44]

$$I = \kappa \int d^2x \sqrt{-g_2} \left[ \eta \left( R_2 + \frac{8}{L^2} \right) - \frac{L^2}{4} F^2 \right], \quad (50)$$

by simply identifying

$$\kappa = \frac{1}{4G_4}, \quad L = -2q, \quad \eta = q^2 = \frac{L^2}{4}. \quad (51)$$

The central charge for (50) was derived in [43, 44] as

$$c = \frac{3}{2} k L^4 E^2, \quad (52)$$

where the electric field is  $E^2 = 4\eta/L^4 = 1/L^2$ . It's easy to see that our results of the central charge,  $c = 6q^2/G_4 = 3L^2/2G_4$  and level  $k = 1/G_4$  are consistent with this relation. Indeed our results for level and right moving central charge can be directly computed via the formulae in [44] (with an opposite sign due to the negative value of  $L$ )

$$k = 4\kappa, \quad c = 24\kappa\eta. \quad (53)$$

Our results indicate that the central charge derived from the viewpoint of  $\text{AdS}_2/\text{CFT}_1$  correspondence is the same as the one derived from the 2D CFT on a strip. This equality seems hint that  $\text{CFT}_1$  can be regarded as an chiral  $\text{CFT}_2$  [45, 46, 47].

## VI. CONCLUSION AND DISCUSSION

In this paper, we reconsidered the quantum gravity description of near horizon extremal RN black hole by using the boundary counterterm approach in the context of  $\text{AdS}_2/\text{CFT}_1$  correspondence. We found that the right moving central charge and the level of the dual 1D CFT are  $c_R = 6q^2/G_4 = 6Q^2$  and  $k = 1/G_4$ , respectively. Since there is no gravitational anomaly, similar to the near extremal Kerr black hole case [35], it is naturally to expect that

the left and right moving central charges are identical. Therefore it seems to have a discrepancy comparing with the previous result of left moving central charge  $c_L = 6Q^3$  obtained from the warped  $\text{AdS}_3/\text{CFT}_2$  prescription in [36, 37]. Regarding to the result i.e.  $c_L = 6Q^3$  in [36, 37], we noticed that authors uplifted the RN black hole into higher dimensions by simply assuming the radius of extra dimensional cycle to be one and its period to be  $2\pi$ . In this approach, however, the size of the extra dimension could affect the result of the central charge and there are no physically sensible preference to determine this size. Although, there is also a normalization scale of AdS radius (negative) to be specified in our approach, but we have more clear physical prescriptions to resolve this scale. Therefore, the result of central charge is supported by several novel properties. Moreover, as a crosscheck, our central charge and level agree with the results obtained from a 2D Maxwell-dilaton quantum gravity which is equivalent to a  $\text{CFT}_2$  on a strip [43]. The agreement of the results from different viewpoints also indicates that  $\text{CFT}_1$  may can be treated as a chiral part of  $\text{CFT}_2$ .

## APPENDIX A: CENTRAL CHARGE FOR DYONIC BLACK HOLES

We can easily generalized our analysis to the RN black holes including magnetic charge. The corresponding 2D effective action is

$$I = \frac{q^2 + p^2}{4G_4} \int d^2x \sqrt{-g_2} \left( e^{-2\psi} R_2 + \frac{2}{q^2 + p^2} - e^{2\psi} \frac{2p^2}{(q^2 + p^2)^2} + 2\nabla_\mu e^{-\psi} \nabla^\mu e^{-\psi} - e^{-2\psi} F^2 \right). \quad (\text{A1})$$

The magnetic charge parameter,  $p$ , adjusts the  $\text{AdS}_2$  radius and the cosmological constant term. One can derive the right moving central charge by repeating the same calculation or simply by the relations eq.(53) with the following values read out from identifying two actions (A1) (taking  $\psi = 0$ ) and (50)

$$\kappa = \frac{1}{4G_4} \frac{q^2}{q^2 + p^2}, \quad \eta = \frac{L^4}{4} = \frac{(q^2 + p^2)^2}{q^2}. \quad (\text{A2})$$

Finally, the central charge is

$$c_R = \frac{6}{G_4} (q^2 + p^2) = 6(Q^2 + P^2). \quad (\text{A3})$$

One novel property is that the central charge also exhibits the electric-magnetic duality which can be apparently seen from the gravity side. However, technically, it is subtle to compute the central charge directly for pure magnetic RN black hole. It's easy to see that when

imposing  $\psi = 0$ , the effective action (A1) becomes pure 2D gravity, i.e. the cosmological constant vanishes and the gauge field disappears. In such circumstance, the effective AdS radius diverges and the level vanishes.

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